Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$\therefore x=12\frac{12}{29}$ days, the time it takes B.

$58x/15=48$ days, the time it takes C.

Solved in like manner by *H. C. WHITAKER*, *P. S. BERG*, *B. F. SINE*, *J. F. TRAVIS*, and *P. H. PHILBRICK*.

NOTE.—This problem was proposed because we held that the wording of the problem was not sufficiently explicit. The fact that our contributors have taken the different views indicated by the above solutions confirms our opinion. The problem has been worded a little different in the last edition of Dr. Milne's Algebra. The wording of the problem is intended to lead to the first solution. B. F. F.

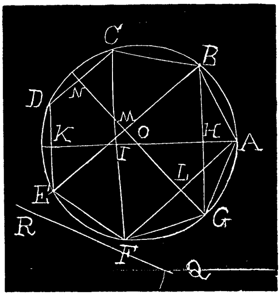
GEOMETRY.

122. Proposed by **G. I. HOPKINS, A. M.**, Professor of Mathematics and Physics, Manchester High School, Manchester, N. H.

If perpendiculars are dropped from the vertices of a regular polygon upon any diameter of the circumscribed circle, the sum of the perpendiculars which fall on one side of this diameter is equal to the sum of those which fall on the opposite side. [From Chauvenet's *Treatise on Elementary Geometry*.]

I. Solution by HENRY HEATON, B. Sc., Atlantic, Ia.

Let $ABCDEFG$ represent a regular heptagon circumscribed by the circle whose center is O. Draw a diameter through A. It will be perpendicular to BG, CF, and DE, and it will bisect them in H, I, and K.



Draw any line RQ and for convenience call the perpendiculars from A, B, C, etc., upon RQ, a, b, c, etc. Then $b+g=2h$, $c+f=2i$, and $d+e=2k$.

Because K, I, H, and A are upon AK it can readily be shown that there is a point P upon AK such that $7p=2k+2i+2h+a=a+b+c+d+e+f+g$, and that this point is independent of the direction of RQ. In a similar manner it may be shown that there is a similar point p' upon the diameter through G, such that $7p'=a+b+c+d+e+f+g$. Hence $p=p'$. But this cannot be true without regard to the direction of RQ unless P and P' are at the intersection of the two diameters, that is, at the center. Hence the perpendicular from O upon any line RQ is one-seventh the sum of the perpendiculars a, b, c, d, e, f, and g. Hence if RQ passes through O, $a+b+c+d+e+f+g=0$.

It is evident that a similar demonstration may be made for any regular polygon.

II. Solution by J. SCHEFFER, A. M., Hagerstown, Md., and CHARLES C. CROSS, Whaleyville, Va.

In a polygon of an even number of sides a diameter drawn through one of the vertices will pass through another vertex which will be symmetrical with reference to the center of the circle as the center of symmetry. Consequently, the sum of the perpendiculars drawn from the vertices on one side of any diameter will be equal to the sum of the perpendiculars drawn from the vertices of the other side of that diameter. If the number of sides of the polygon is uneven there will be no such symmetry.

Let MN be a diameter, $ABCDEFG$ be a regular polygon of n sides, n being an odd number, and O the center. Draw all the radii to the vertices, then $\angle ACB = \angle BOC = \angle COD = \ldots\ \ldots = 2\pi/n = \beta$. Denote $\angle AOM$ by a, $\angle GOM$ by a'. It will be perceived at once that on one side of the diameter MN there are $\frac{1}{2}(n-3)$ triangles, and on the other $\frac{1}{2}(n-1)$ triangles, not cut by the diameter.

Employing the well-known formula of the difference of two cosines, we have

$$\cos(a-\tfrac{1}{2}\beta)-\cos(a-\tfrac{1}{2}\beta)=2\sin a\sin\beta,$$

$$\cos(a+\tfrac{1}{2}\beta)-\cos(a+\tfrac{3}{2}\beta)=2\sin(a+\beta)\sin\tfrac{1}{2}\beta,$$

$$\cos(a+\tfrac{3}{2}\beta)-\cos(a+\tfrac{5}{2}\beta)=2\sin(a+2\beta)\sin\tfrac{1}{2}\beta$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$\cos[a+\tfrac{1}{2}(2m-1)\beta]-\cos[a+\tfrac{1}{2}(2m+1)\beta]=\pm2\sin(a+m\beta)\sin\tfrac{1}{2}\beta.$$

Adding,

$$\frac{\sin(a+\frac{1}{2}m\beta)\sin\frac{1}{2}(m+1)\beta}{\sin\frac{1}{4}\beta}=\sin a+\sin(a+\beta)+\sin(a+2\beta)+\ldots\ldots+\sin(a+m\beta).$$

Putting the radius of the circle=1, this last formula gives us the sum of the perpendiculars drawn from the vertices upon MN on either side of the latter.

Putting $m=\frac{1}{2}(n-3)\beta$, this sum

$$=\frac{\cos(a_1-\frac{3}{4}\beta)\cos\frac{1}{4}\beta}{\sin\frac{1}{2}\beta}.$$

Putting $m=\frac{1}{2}(n-1)$ we get

$$\frac{\cos(a_1-\frac{1}{4}\beta)\cos\frac{1}{4}\beta}{\sin\frac{1}{2}\beta},$$

but $a_1=\beta-a$, therefore the latter expression is

$$\frac{\cos(\frac{3}{4}\beta-a)\cos\frac{1}{4}\beta}{\sin\frac{1}{2}\beta}.$$

Consequently we have equality for the two sums of the perpendiculars, which proves the theorem.

Also solved by *G. B. M. ZERR*.

123. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

A etant le point d' intersection. des médianes d' un triangle ABC, démontrerque $AB^2+BC^2+CA^2=3(GA^2+GB^2+GB^2)$. [Ex. 84, Géométrie. No. 2, 1^{re} Anne *L' Éducation Mathématique.*]